LONG-RANGE WAKEFIELDS IN DEFLECTING CAVITIES

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The basic equations for longitudinal and transverse wakefields of a point charge in a resonator are:

$$w_{//}(t) = 2k_{//}e^{-t/\tau}\left[\cos(\overline{\omega}t) - \frac{\sin(\overline{\omega}t)}{\overline{\omega}\tau}\right]$$

$$w_{\perp}(t) = \frac{2c}{\overline{\omega}}\frac{k_{\perp}}{b^2}e^{-t/\tau}\sin(\overline{\omega}t)$$

where

$$\tau = \frac{2Q}{\omega} \ \textit{and} \ \overline{\omega} = \sqrt{\omega^2 - 1/\tau^2}$$

From a list of the Q's, resonant frequencies and loss factors for all the deflecting cavities high order modes it is possible to calculate the total long-range wakefield simply by summing up each individual mode contribution.

Because of the high Q values in superconducting cavities, and our 10 kHz repetition rate, the wakefield generate by a bunch passage doesn't decay fast enough to become negligible when the next bunch passes through the cavity. The total wakefield builds up and eventually reaches a steady-state value which we are interested into knowing.

Longitudinal wake

First, let's suppose that all the monopole modes up to the beam pipe cut-off have Q = 10^10.

The wake from a single bunch decays quite slowly (Fig.1)

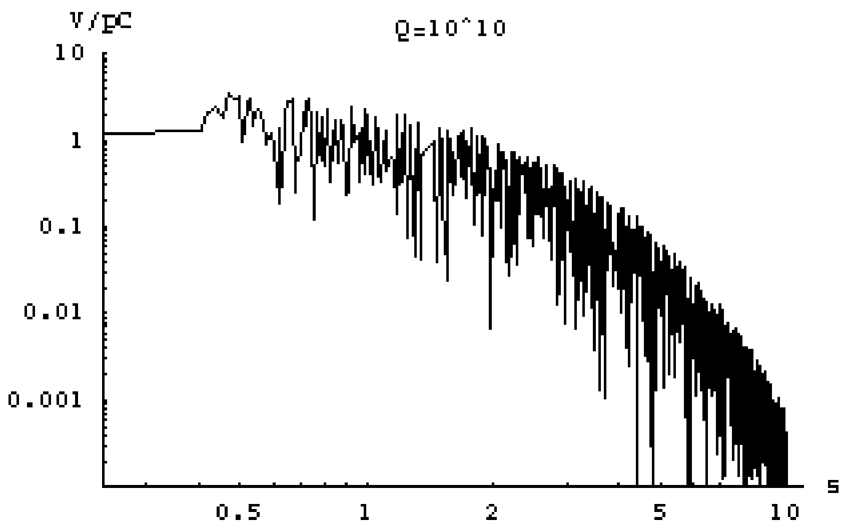

Fig.1 Longitudinal wake.

The steady-state value of 85.5 kV/pC is reached after about 50000 bunch passages, i.e. 5 seconds.

Since there are 4 cavities and we are dealing with 1 nC of charge, the total steady-state wakefield would be around 342 MV which correspond to a 13.7% energy spread.

The steady-state value is proportional to the HOM Q's, therefore damping the cavities in such way to have Q=10^9 would reduce the above values tenfold.

From the loss factor list we can see that the two modes at 2.8581 and 2.8685 GHz have a dominating effect. We can check how the wakefield is reduced when damping those two modes only to Q=10^9, leaving all the others unchanged.

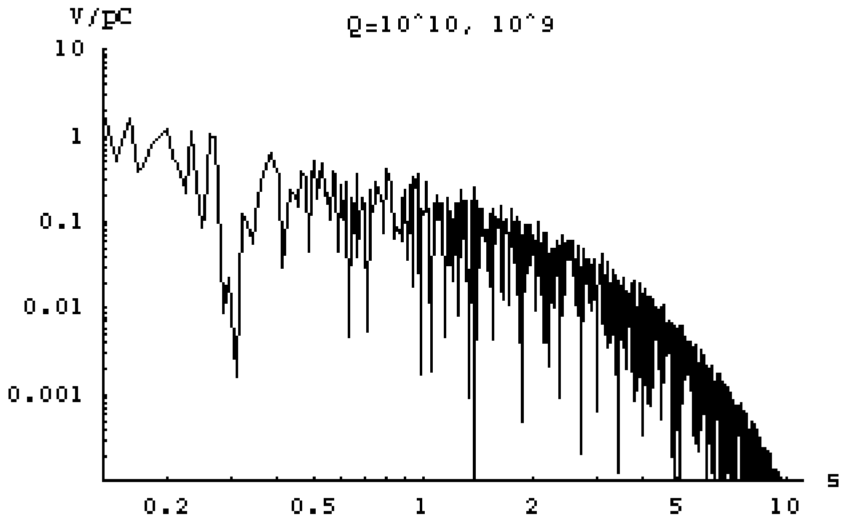

Fig.2 Longitudinal wake, two highest loss factor modes damped by a factor 10.

Comparing Figs.1 and 2 we can see that a small damping of those two modes already has a noticeable effect.

The wakefield steady-state value is 21.9 kV/pC, i.e. it is reduced by a factor 4. This is consistent with a simple theoretical model which assigns a loss factor of about 70 kV/pC to the two dominating modes when their Q = 10^10.

Transverse wake

The transverse wake from dipole modes is dominated by the deflecting mode. If we take that out and consider a Q = 10^10 for all the HOM, we obtain the single bunch wake of Fig.3

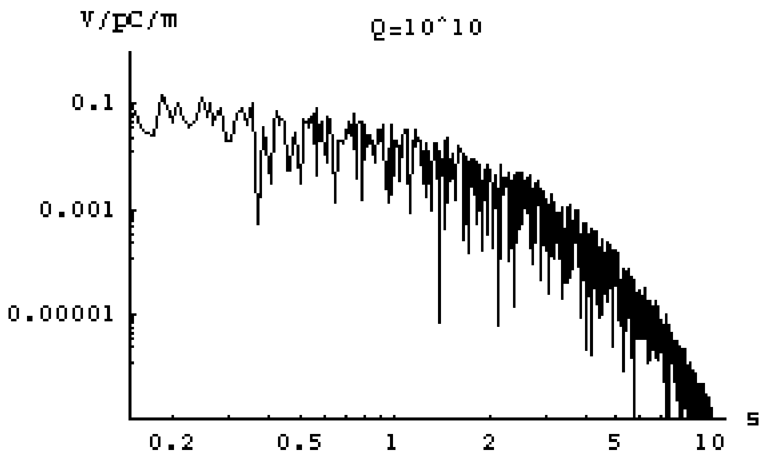

Fig.3 Transverse wake.

The transverse wake is zero at the passage of the point charge and raises approximately linearly shortly after that; we consider its value after 0.5 ps, which is somehow representative for a 2 ps long bunch. The steady-state value for the transverse wake calculated this way is 26.2 V/pC/m, in this case too this value is directly proportional to the HOM quality factor.